MAXIMUM ENTROPY APPROXIMATION

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Abstract

Data approximation from scattered points in $\mathbf{R}^d$ is required in many applications: computer graphics and visualization, regression models, image processing, and numerical methods to name a few. In this paper, the construction of scattered data approximants is studied using the principle of maximum entropy [1]. For under-determined and ill-posed problems, the Shannon-Jaynes principle of maximum information-theoretic entropy [1] provides the least-biased solution when insufficient information is available. Consider a function $u(\mathbf{x})$ that is approximated by a linear combination of shape functions $\{\phi_i\}$: $u^h(\mathbf{x}) = \sum_{i=1}^n \phi_i(\mathbf{x}) u_i$. We establish a link between maximizing entropy and data approximation by viewing the shape functions $\{\phi_i\}$, which form a partition of unity, as a discrete probability distribution. The nodal coordinates $\mathbf{x}_i$ are the events, and given a point p with coordinate $\mathbf{x}$, the shape function value is the "probability of influence of the node i at p" [2]. The ϕ_i are computed by maximizing the uncertainty $H(\boldsymbol{\phi}) = -\sum_{i=1}^n \phi_i \log \phi_i$, subject to the linear reproducing conditions: $\sum_{i=1}^n \phi_i = 1$, $\sum_{i=1}^n \phi_i x_i = x$, $\sum_{i=1}^n \phi_i y_i = y$. This formulation bears similarity to the Linear Interpolation Maximum Entropy (LIME) algorithm used in supervised learning [3]. In the proposed approach, only the nodal coordinates are used, and neither the nodal connectivity nor any user-defined parameters are required to determine ϕ_i—the defining characteristics of a *meshfree Galerkin approximant* [4]. The ϕ_i are computed using a convex minimization algorithm, and comparisons are drawn with well-known polygonal finite element interpolants [5]. The use of Bayesian methods and information-theoretic principles in materials and mechanics computations—inverse problems, topology optimization and material design, multiscale modeling, and meshfree methods, holds promise.

References:

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